CHAPTER 1

ELECTRIC CHARGES AND FIELDS

Electrostatics deals with the study of forces, fields and potentials arising from static charges.

ELECTRIC CHARGE
Electric charge is the property of matter that exhibits its electrostatic interaction with other matter.

Features of electric charges:

- Unlike charges attract and like charges repel each other.
- The material, which loses electrons or has a deficit of electrons, becomes positively charged.
- The material, which gains electrons or has excess electrons, becomes negatively charged.
- Charge is a scalar quantity.

A **gold leaf electroscope** is used to determine whether a body is charged or uncharged.

Properties of electric charge:

1. Additivity of charges:

If a system contains n point charges q_1,... q_n, the total charge of the system is obtained simply by adding algebraically q_1 to q_n ,

$$q = q_1 + q_2 + \cdots + q_n$$

2. Total charge in an isolated system is always conserved.

3. Quantization of charge: charge on an object is always an integral multiple of the basic unit of charge,

i.e., $$q = ne$$

- Conductors are materials that readily allow the flow of electric current through them.
- Insulators are materials that offer high resistance to the flow of electricity through them.
- Charges are distributed over the entire surface of a conductor, whereas they stay in one place in the case of an insulator.

Grounding: The process of transforming any excess charge on a body to the earth, using a wire connected to the earth, is known as earthing or grounding.

Charging a body
There are three, methods to charge a body:
1. Charging by friction
2. Charging by conduction
3. Charging by induction

1. The process of charging two uncharged or neutral objects made of different materials by rubbing against each other is called **charging by friction**.
2. The process of charging by bringing a charged body in contact with a neutral body is called **charging by conduction**.
3. The process of polarization of the charge on an uncharged body when a charged body is held close to it is called **induction of charge**.

Coulomb's Law
Its states that the electrostatic force between two charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between the charges.

$$F = k \frac{|q_1 \ q_2|}{r^2}$$

$k = 1/4\pi\varepsilon_0$

where, ε_0 is called the *Permittivity of Free Space*.
The value of ε_0 in **SI units** is

$$\boldsymbol{\varepsilon_0 = 8.854 \times 10^{-12}\ C^2\ N^{-1} m^{-2}}$$

Permittivity of Free Space

It represents the capability of a vacuum to permit electric fields. It is also connected to the energy stored within an electric field and capacitance.

Dielectric Constant

The ratio of the permittivity of the medium to the permittivity of the free space

mathematically expressed as:

$$\boldsymbol{k = \frac{\varepsilon}{\varepsilon_0}}$$

Where,

- κ is the dielectric constant
- $\boldsymbol{\varepsilon}$ is the permittivity of the substance
- $\boldsymbol{\varepsilon_0}$ is the permittivity of the free space

FORCES BETWEEN MULTIPLE CHARGES

PRINCIPLE OF SUPERPOSITION

Force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges.

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots + \mathbf{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13} + \ldots + \frac{q_1 q_n}{r_{1n}^2}\hat{\mathbf{r}}_{1n}\right]$$

The vector sum is obtained as usual by the parallelogram law of addition of vectors. All of electrostatics is basically a consequence of Coulomb's law and the superposition principle.

ELECTRIC FIELD

The concept of field was first introduced by Faraday

- A charge Q produces an electric field everywhere in the surrounding.
- Electric field (E) is the force on unit positive charge acting at a particular point in the electric field.

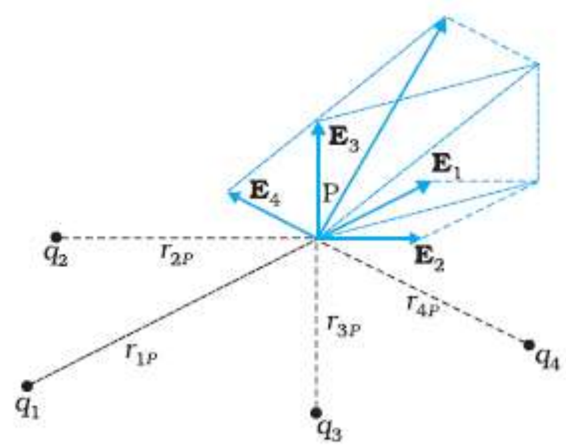
$$\mathbf{E} = \lim_{q\to 0}\left(\frac{\mathbf{F}}{q}\right)$$

- Electric field, E, is a vector quantity pointing in the direction of the force on a unit positive charge at a point in the electric field.

Electric field due to a system of charges

Consider a system of charges $q_1, q_2, \ldots, q_n$ with position vectors $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n$ relative to some origin O.

By the superposition principle, the electric field **E** at **r** due to the system of charges is



$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_1(\mathbf{r}) + \mathbf{E}_2(\mathbf{r}) + \ldots + \mathbf{E}_n(\mathbf{r})$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}^2}\hat{\mathbf{r}}_{1P} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}^2}\hat{\mathbf{r}}_{2P} + \ldots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{nP}^2}\hat{\mathbf{r}}_{nP}$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{iP}^2}\hat{\mathbf{r}}_{iP}$$

Physical significance of electric field

- Electric field is a characteristic of the system of charges and is independent of the test charge that you place at a point to determine the field.
- The term *field* in physics generally refers to a quantity that is defined at every point in space and may vary from point to point.
- Electric field is a vector field, since force is a vector quantity.

The accelerated motion of charge q_1 produces electromagnetic waves, which then propagate with the speed ***c****, reach q_2 and cause a force on q_2.*

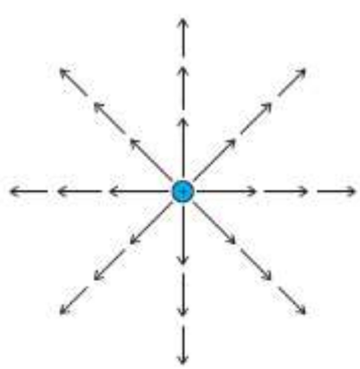
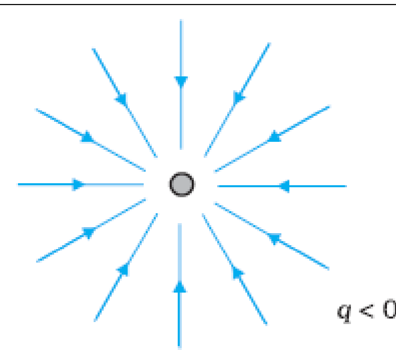
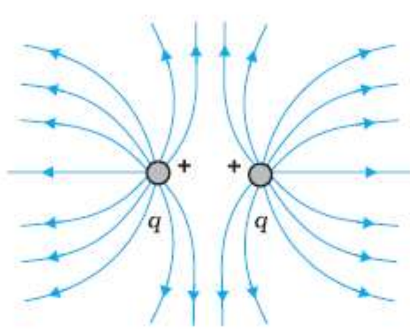
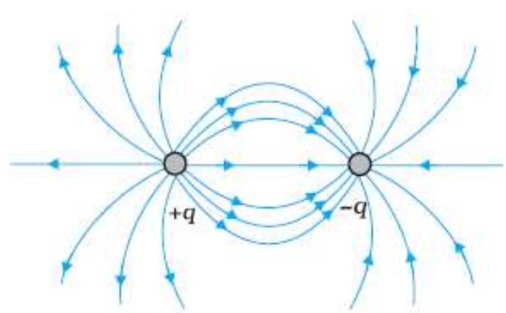
ELECTRIC FIELD LINES

Electric field lines are thus a way of pictorially mapping the electric field around a configuration of charges.

The field lines follow some important general properties:

a. Field lines start from positive charges and end at negative charges. If there is a single charge, they may start or end at infinity.
b. In a charge-free region, electric field lines can be taken to be continuous curves without any breaks.
c. Two field lines can never cross each other, if they do there will be two directions at the point of intersection which is not possible.
d. Electro*static* field lines do not form any closed loops. This follows from the conservative nature of electric field.

Electric Field lines for:-

1. POSITIVE CHARGE	2. NEGATIVE CHARGE
	
3. TWO POSITIVE CHARGES	4. DIPOLE
	

CONTINUOUS CHARGE DISTRIBUTION

The continuous charge distribution system is a system in which the charge is uniformly distributed over the conductor. In continuous charge system, infinite numbers of charges are closely packed and have minor space between them. Unlikely from the discrete charge system, the continuous charge distribution is uninterrupted and continuous in the conductor. There are three types of the continuous charge distribution system.

1. Linear Charge Distribution
2. Surface Charge Distribution
3. Volume Charge Distribution

1. Linear Charge Density

When the charges are uniformly distributed over the length of a conductor, it is called linear charge distribution. It is also called linear charge density and is denoted by the symbol **λ** (Lambda).

Mathematically linear charge density is

$$\lambda = \frac{dq}{dl}$$

The unit of linear charge density is **C/m**.

2. Surface Charge Density

When the charges are uniformly distributed over the surface of the conductor, it is called Surface Charge Density or Surface Charge Distribution. It is denoted by the symbol **σ** (sigma) symbol and is the unit is **C/m^2**.

It is also defined as charge/ per unit area. Mathematically surface charge density is

$$\sigma = \frac{dq}{ds}$$

where dq is the small charge element over the small surface **ds**. So, the small charge on the conductor will be

$$dq = \sigma\, ds$$

3. Volume Charge Density

When the charges are distributed over a volume of the conductor, it is called Volume Charge Distribution. It is denoted by symbol **ρ** (rho).

In other words, charge per unit volume is called Volume Charge Density and its unit is **C/m^3.**

Mathematically, volume charge density is $$\rho = \frac{dq}{dv}$$

where **dq** is small charge element located in small volume **dv**.

Electric dipole

- An electric dipole comprises of a positive charge and a negative charge separated by a finite distance.
- Electric dipole moment, $\vec{p}$, is the product of the length of the dipole (2a) and the magnitude of the charge (q),

$$\vec{p} = q2a$$

- Molecules in which the centers of all positive and negative charges coincide are called **non- polar molecules**. Non polar molecules do not have any dipole moment.
- Molecules in which the centers of all positive and negative charges do not coincide are called **polar molecules**. Polar molecules are permanent dipoles.

Electric field due to a dipole

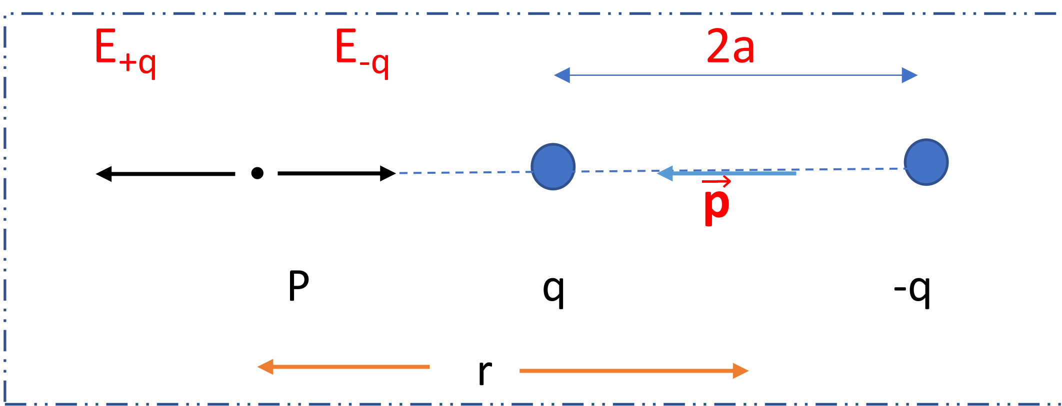
The electric field of the pair of charges (–q and q) at any point in space can be found out from Coulomb's law and the superposition principle.
Following two cases:

(i) when the point is on the dipole axis, and

(ii) when it is in the equatorial plane of the dipole, i.e., on a plane perpendicular to the dipole axis through its centre.

(iii) The electric field at any general point P is obtained by adding the electric fields E–q due to the charge –q and E+q due to the charge q, by the parallelogram law of vectors.

(i) For points on the axis



P be at distance *r* from the center of the dipole on the side of the charge *q*,

$$\vec{E}_{-q} = -\frac{q}{4\pi\epsilon_o(r+a)^2}\hat{p} \quad , \quad \vec{E}_{+q} = \frac{q}{4\pi\epsilon_o(r-a)^2}\hat{p}$$

$\hat{p}$ is the unit vector along the dipole axis (from –*q* to *q*).

The total field at P is

$$\vec{E} = \vec{E}_{+q} + \vec{E}_{-q} \implies \vec{E} = \frac{q}{4\pi\epsilon_o}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

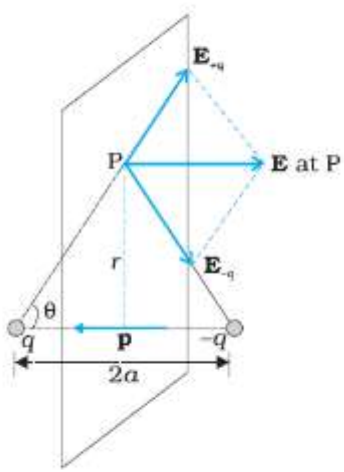
$$\vec{E} = \frac{q}{4\pi\epsilon_o}\left[\frac{(r^2 + a^2 + 2ar) - (r^2 + a^2 - 2ar)}{\{(r-a)(r+a)\}^2}\right]\hat{p}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_o}\left[\frac{4ar}{(r^2 - a^2)^2}\right]\hat{p}$$

For *r* >> *a*

$$\boxed{\vec{E} = \frac{4qar}{4\pi\epsilon_o r^3}\hat{p} \quad \Rightarrow \quad \vec{E} = \frac{2\vec{p}}{4\pi\epsilon_o r^3}\hat{p}}$$

(ii) For points on the equatorial plane



The magnitudes of the electric fields due to the two charges $+q$ and $-q$ are given by

$$E_{+q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2 + a^2}$$

$$E_{-q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2 + a^2}$$

the components normal to the dipole axis cancel away. The components along the dipole axis add up.

The total electric field is opposite to $\hat{p}$. We have

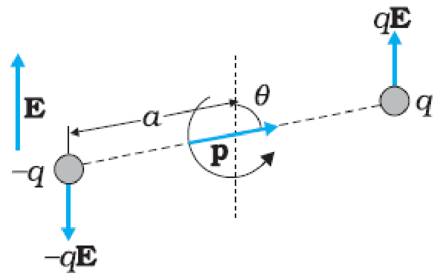
$$\vec{E} = -(\vec{E}_{+q} + \vec{E}_{-q})\cos\theta\ \hat{p}$$

$$\vec{E} = -\frac{2q}{4\pi\epsilon_o(r^2 + a^2)}\cos\theta\ \hat{p}$$

$$\cos\theta = \frac{qO}{qP} = \frac{a}{\sqrt{r^2 + a^2}}$$

$$\vec{E} = -\frac{2q}{4\pi\epsilon_o(r^2 + a^2)} \cdot \frac{a}{\sqrt{r^2 + a^2}}\,\hat{p} \quad => \vec{E} = \frac{\vec{p}}{4\pi\epsilon_o r^3}\,\hat{p}$$

Dipole in an external uniform Electric Field



Consider a permanent dipole of dipole moment p in a uniform external field E. There is a force qE on q and a force –qE on –q.

The net force on the dipole is zero, since E is uniform. However, the charges are separated, so the forces act at different points, resulting in a torque on the dipole.

When the net force is zero, the torque (couple) is independent of the origin.

Magnitude of torque = q E × 2 a sinθ

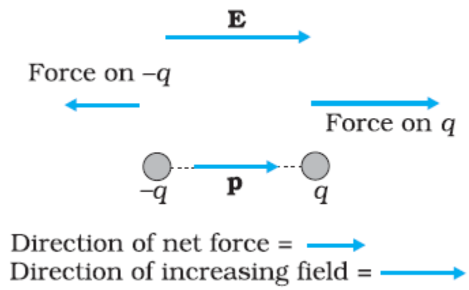
= 2 q a E sinθ

Its direction is normal to the plane of the paper, coming out of it.
Or

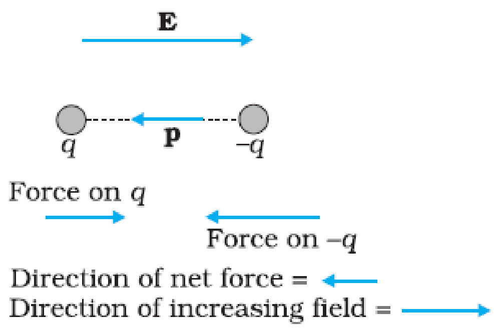
$$\vec{\tau} = \vec{p} \times \vec{E}$$

Dipole in Non - Uniform External Field
The net force will be non-zero. There will be a torque on the system.

There are two cases: -
(i) If p is parallel to E - when p is parallel to E, the dipole has a net force in the direction of increasing field.



(ii) If p is Antiparallel to E - When p is antiparallel to E, the net force on the dipole is in the direction of decreasing field.



- In either case, the net torque is zero, but there is a net force on the dipole if E is not uniform.
- In general, the force depends on the orientation of p with respect to E.

ELECTRIC FLUX

Electric flux is the number of field lines crossing any area, placed normal to the field at a point is a measure of the strength of electric field at that point.

In the SI system, electric flux is measured in N m^2 C^{-1}

Mathematically,

$$\phi_E = \oint \vec{E}.\vec{dS}$$

Gauss' law

It states that

"The electric flux through any closed surface is proportional to the enclosed electric charge."

$$\phi_E = \frac{q_{net}}{\varepsilon_o}$$

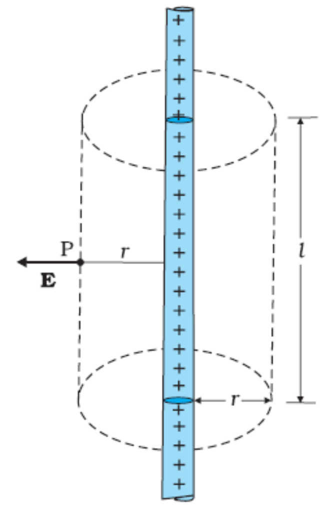
- The net electric flux (ϕ_E) through any closed surface is independent of the shape of the closed surface.

- The net flux through a closed surface not enclosing any charge is zero. Gauss' law is applicable to any closed surface of any arbitrary shape.

Applications of Gauss' law

1. Field due to an infinitely long straight uniformly charged wire

- Imagine a cylindrical Gaussian surface.
- Since the field is everywhere radial, flux through the two ends of the cylindrical Gaussian surface is zero.
- At the cylindrical part of the surface, **E** is normal to the surface at every point.



The surface area of the curved part $\overrightarrow{ds}$= $2\pi rl$,

where l is the length of the cylinder.

Flux through the Gaussian surface ϕ_E

ϕ_E = flux through the curved cylindrical part of the surface

$$\phi_E = \vec{E}\,.\overrightarrow{ds} = E \times 2\pi rl$$
$$q = \lambda\, l.$$

Gauss's law then gives

$$E \times 2\pi rl = \frac{\lambda l}{\varepsilon_o}$$

i.e.,

$$E = \frac{\lambda}{2\pi r\varepsilon_o}$$

Vectorially,

$$\vec{E} = \frac{\lambda}{2\pi r\varepsilon_o}\hat{n}$$

where $\hat{n}$ is the radial unit vector in the plane normal to the wire passing through the point. $\vec{E}$ is directed outward if λ is positive and inward if λ is negative.

2. Field due to a uniformly charged infinite plane sheet

- Let σ = uniform surface charge density of an infinite plane sheet.
- Take the x-axis normal to the given plane.
- Taking the Gaussian surface to be a rectangular parallelepiped of cross-sectional area A.
- Only the two faces 1 and 2 will contribute to the flux;
- electric field lines are parallel to the other faces and they, therefore, do not contribute to the total flux.

Therefore, the net flux through the Gaussian surface, $\phi_E = \vec{E}\,.\overrightarrow{ds} = 2EA$.

The charge enclosed by the closed surface,

$$q = \sigma A$$

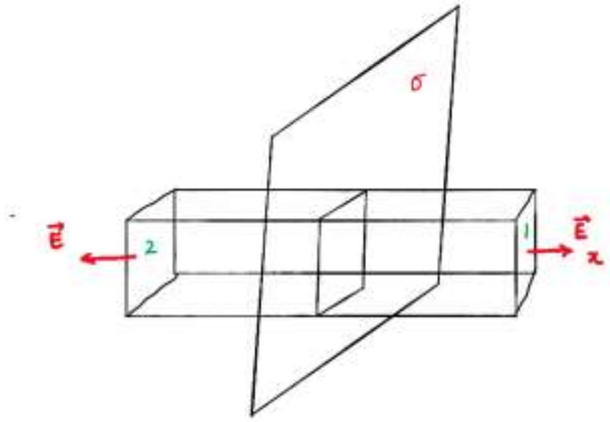
By Gauss's law,

$$\phi_E = \vec{E}\,.\overrightarrow{ds} = \frac{q}{\varepsilon_o}$$

$$2EA = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{2\varepsilon_o}$$

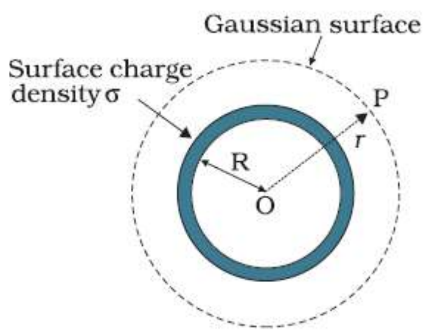
$$\vec{E} = \frac{\sigma}{2\varepsilon_o}\hat{n}$$



where $\hat{n}$ is a unit vector normal to the plane and going away from it.
$\vec{E}$ is directed away from the plate if σ is positive and toward the plate if σ is negative.

3. Field due to a uniformly charged thin spherical shell

Let σ be the uniform surface charge density of a thin spherical shell of radius R.



(i) Field outside the shell:

- Consider a point P outside the shell with radius vector **r**.
- Taking the Gaussian surface to be a sphere of radius r and with centre O, passing through P.
- $\vec{E}$ at each point of the Gaussian surface has the same magnitude and is along the radius vector at each point.

The flux through the Gaussian surface, $\phi_E = \vec{E}\,.\overrightarrow{ds} = E.4\,\pi\,r^2$

The charge enclosed, $q = \sigma\ 4\,\pi\,R^2$
By Gauss's law

$$\phi_E = \vec{E}\,.\overrightarrow{ds} = \frac{q}{\varepsilon_o}$$

$$E.4\,\pi\,r^2 = \frac{\sigma\ 4\,\pi\,R^2}{\varepsilon_o}$$

$$E = \frac{\sigma R^2}{\varepsilon_o r^2}$$

(ii) ***Field on the surface of the shell:***

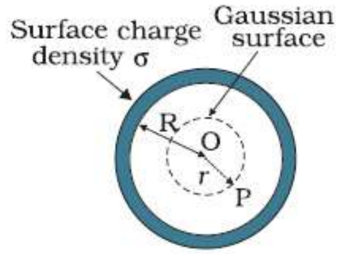
On the surface of the shell , $R = r$

Therefore from the equation

$$E = \frac{\sigma R^2}{\varepsilon_o r^2}$$

$$\Rightarrow E = \frac{\sigma}{\varepsilon_o}$$

(iii) Field inside the shell:



In this case, the Gaussian surface encloses no charge. Gauss's law then gives

$$E \,.\, 4\,\pi\, r^2 = 0$$

i.e.,

$$E = 0 \qquad , \qquad (r < R)$$

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References:

1. Physics Vol. I by NCERT
2. Fundamental of Physics by Resnik & Halliday